

Super Yang-Mills Theory as a Twistor Matrix Model

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We introduce a covariant finite regulator for $\mathcal{N} = 4$ super Yang-Mills theory on S^4 . Our formulation is based on holomorphic Chern-Simons theory on twistor space. By switching on a large background flux, the twistor space dissolves into a fuzzy geometry, with a finite number of points. The large N continuum limit of the matrix model naturally approaches ordinary $\mathcal{N} = 4$ SYM. We comment on the relation between our model and the 4D quantum Hall effect.

Introduction

Considerations from holography suggest that quantum field theory involves an overcomplete number of degrees of freedom. It is believed that string theory somehow avoids this overcounting, as evidenced by the existence of dual formulations such as AdS/CFT. Even so, the microscopic origin of holography remains poorly understood, especially in the context of space-times with positive curvature. One way to try to make some progress with this question is to look for self-consistent covariant ways of regulating gauge theories with holographic duals.

In this short note, we present a concrete proposal for a covariant regulator of $U(N_c)$ $\mathcal{N} = 4$ super Yang-Mills theory (SYM) on S^4 in terms of a non-commutative, or “fuzzy” twistor space with a finite number of fuzzy points. The theory is obtained via a decoupling limit from ordinary holomorphic Chern-Simons (hCS) theory on super-twistor space with a large background flux switched on. The presence of the flux fuzzifies the space, and also generates a BC system localized on $D6/D2$ -brane intersections wrapping the twistor lines. Via the twistor correspondence, the large N continuum limit of the matrix model naturally reproduces ordinary $\mathcal{N} = 4$ SYM theory on S^4 .

Twistor Gauge Theory

Twistors (see [1] for a review) provide a convenient way to characterize massless theories. In complexified Minkowski space \mathbb{M} , light-like momenta $p_{AA'}$ can be represented as a product $p_{AA'} = \tilde{\pi}_A \pi_{A'}$ of a chiral and an anti-chiral spinor. Given a space-time point $x^{AA'}$, we can define a corresponding spinor $\omega^A = ix^{AA'} \pi_{A'}$, and thereby obtain a set of homogeneous coordinates $Z^\alpha = (\omega^A, \pi_{A'})$ on projective twistor space \mathbb{PT}^\bullet . The dual twistor space, with coordinates $\tilde{Z}_\beta = (\tilde{\pi}_A, \tilde{\omega}^{A'})$, is denoted by \mathbb{PT}_\bullet . Geometrically, \mathbb{PT}^\bullet and its dual both define a \mathbb{CP}^3 .

A basic feature of twistor theory is that physical space-time is derived from twistor data. The two equations

$$\omega^A = ix^{AA'} \pi_{A'} \quad (1)$$

for $A=1,2$ cut out a \mathbb{CP}^1_x in \mathbb{PT}^\bullet , which is the twistor line associated with a single point $x^{AA'}$ in \mathbb{M} . Conversely, a

point in twistor space corresponds to a null-plane in \mathbb{M} .

This correspondence can be extended to superspace coordinates $\mathcal{X}^{II'} = (x^{AA'} | \theta^{A'i})$, and supertwistor coordinates $\mathcal{Z}^I = (\omega^A, \pi_{A'} | \psi^i)$. The resulting supertwistor space $\mathbb{CP}^{3|4}$ is a Calabi-Yau manifold, a natural target space for the topological string [2]. The open string sector of the B-model is holomorphic Chern-Simons theory

$$S_{hCS} = \int_{\mathbb{CP}^{3|4}} \Omega \wedge \text{Tr} \left(\mathcal{A} \bar{\partial} \mathcal{A} + \frac{2}{3} \mathcal{A}^3 \right) \quad (2)$$

where $\mathcal{A} = \mathcal{A}_I d\bar{Z}^I$ denotes a gauge superfield. This theory is physically equivalent to self-dual $\mathcal{N} = 4$ SYM [2], provided that \mathcal{A} is restricted to be flat over each twistor line. Each component of the holomorphic superfield \mathcal{A}_I corresponds to a field of the 4D SYM theory.

Twistor Quantization

The product $\mathbb{PT}^\bullet \times \mathbb{PT}_\bullet$ is naturally viewed as a quantized phase space [4]. The twistor commutation algebra

$$[Z^\alpha, \tilde{Z}_\beta] = \hbar \delta^\alpha_\beta \implies \left\{ \begin{array}{l} [\omega^A, \tilde{\pi}_B] = \hbar \delta_B^A \\ [\pi_{A'}, \tilde{\omega}^{B'}] = \hbar \delta_{A'}^{B'} \end{array} \right\}. \quad (3)$$

provides a representation of $sl(4, \mathbb{C})$, the algebra of the complexified conformal group with generators:

$$\begin{aligned} P_{AA'} &= \tilde{\pi}_A \pi_{A'} \quad , \quad K_{AA'} = \tilde{\omega}_{A'} \omega_A \quad , \\ J_{AB} &= \tilde{\pi}_{(A} \omega_{B)} \quad , \quad \tilde{J}_{A'B'} = \tilde{\omega}_{(A'} \pi_{B')} \quad , \\ D &= \frac{1}{2} (\tilde{\pi}^A \omega_A - \tilde{\omega}_{B'} \pi^{B'}). \end{aligned} \quad (4)$$

The extension to space-time supersymmetric theories is achieved by including four fermionic oscillators ψ^i with $\{\psi^i, \tilde{\psi}_j\} = \hbar \delta_j^i$, which then provides a representation of $sl(4|4)$. In what follows we shall often leave implicit the extension to the fermionic case.

Specifying a real space-time signature amounts to a choice of realization of $sl(4, \mathbb{C})$, *i.e.* a convention for Hermitian conjugation. The three signatures $(+++)$, $(++--)$ and $(-+++)$ correspond to the realizations of $sl(4, \mathbb{C})$ respectively by $su(4)$, $sl(4, \mathbb{R})$ and $su(2, 2)$. Ex-

plicit realizations of each case are:

$$su(4) : \quad \omega_A^\dagger = \tilde{\pi}_A, \pi_{B'}^\dagger = \tilde{\omega}_{B'} \quad \text{and} \quad \hbar \in \mathbb{R} \quad (5)$$

$$su(2, 2) : \quad \omega_A^\dagger = \tilde{\omega}_{A'}, \pi_{A'}^\dagger = \tilde{\pi}_A \quad \text{and} \quad \hbar \in \mathbb{R} \quad (6)$$

$$sl(4, \mathbb{R}) : \quad \left\{ \begin{array}{l} \omega_A^\dagger = \omega_A, \pi_{A'}^\dagger = \pi_{A'} \\ \tilde{\omega}_{A'}^\dagger = \tilde{\omega}_{A'}, \tilde{\pi}_{A'}^\dagger = \tilde{\pi}_{A'} \end{array} \right\} \quad \text{and} \quad \hbar \in i\mathbb{R} \quad (7)$$

The reality condition on \hbar is enforced by Hermitian conjugation on the commutators.

In the remainder of this note we restrict to Euclidean signature. The hermitian conjugation map (5) provides an identification $(\mathbb{PT}_\bullet)^\dagger = \mathbb{PT}^\bullet$ which acts on the conformal symmetry generators as:

$$P^\dagger = K, J^\dagger = J, \tilde{J}^\dagger = \tilde{J}, D^\dagger = -D, \quad (8)$$

leaving the $SO(5)$ generators J, \tilde{J} and $\frac{1}{2}(P + K)$ as the only hermitian charges. The Euclidean theory is therefore most naturally viewed as a radially quantized theory on S^4 . The hermitian charges for the supersymmetric case are the symmetries of the supersphere $S^{4|8}$.

The projection $\mathbb{CP}^3 \rightarrow S^4$ is achieved by introducing the quaternionic coordinates

$$(Q_1, Q_2) = (Z_1 + jZ_2, Z_3 + jZ_4) \in \mathbb{HP}^1 = S^4 \quad (9)$$

with \mathbb{HP}^1 the quaternionic projective plane, and j a quaternion. We will make important use of this quaternionic perspective momentarily.

We see that the commutator algebra (3) naturally identifies the twistor coordinates with creation and annihilation operators, and functions on \mathbb{PT} with linear operators on the associated Fock space. Introduce a vacuum state such that $Z^\alpha |\text{vac}\rangle = \psi^i |\text{vac}\rangle = 0$. The Fock space is:

$$\mathcal{F} = \text{span}_{\mathbb{C}} \left\{ \prod_{\beta, i} \tilde{Z}_\beta^{n_\beta} \tilde{\psi}_i^{n_i} |\text{vac}\rangle \right\}. \quad (10)$$

This space is graded by the homogeneity operator:

$$\mathfrak{D} = \tilde{Z}_\beta Z^\beta + \tilde{\psi}_i \psi^i. \quad (11)$$

The level N subspace

$$\mathfrak{D} |\Psi\rangle = \hbar N |\Psi\rangle \quad (12)$$

describes the Hilbert space $\mathcal{H}_{\mathbb{PT}}(N)$ of fuzzy points on a non-commutative $\mathbb{CP}^{3|4}$ with Kähler parameter $\hbar N$. The bosonic subspace of $\mathcal{H}_{\mathbb{PT}}(N)$ consists of the N -fold symmetric product of the fundamental of $sl(4, \mathbb{C})$, and thus fills out an irreducible representation of dimension

$$k = (N+1)(N+2)(N+3)/6. \quad (13)$$

The analogue of the twistor equation in $\mathcal{H}_{\mathbb{PT}}$ is

$$(\omega^A - ix^{AA'} \pi_{A'}) |q_x\rangle = 0, \quad (14)$$

where $x^{AA'}$ is a complex 2×2 matrix defining a point in complexified space-time. Eq. (14) projects onto a fuzzy \mathbb{CP}^1 inside \mathbb{CP}^3 . We refer to this subspace as $\mathcal{H}_x \subset \mathcal{H}_{\mathbb{PT}}$. Each subspace \mathcal{H}_x has dimension $N+1$ and transforms as a spin $j = N/2$ representation of $sl(2, \mathbb{C})$.

Note that the collection of all position eigenstates $||x\rangle$ forms an overcomplete basis. Pick a state $|q_x\rangle \in \mathcal{H}_x$ and an $SO(5)$ rotation $R_{y,x}$ which maps x to y . We can compute the overlap $((y||x)) = \langle q_x | R_{y,x} | q_x \rangle$. In terms of the rotation angle $\theta_{x,y}$ from x to y

$$((y||x)) = (\cos \frac{1}{2} \theta_{x,y})^N. \quad (15)$$

At large N this is a sharply peaked Gaussian function of width $\sigma = 2/\sqrt{N}$.

The Hilbert space $\mathcal{H}_{\mathbb{PT}}$ is closely related to the realization of the quantum Hall effect (QHE) on S^4 given in [5]. In particular, the dimension of $\mathcal{H}_{\mathbb{PT}}$ matches with that of the lowest Landau level (LLL) found in [5].

A Decoupling Limit

To obtain the twistor matrix model, we start with holomorphic Chern-Simons theory on commutative super twistor space. The idea is to turn on a suitable gauge bundle with a large instanton charge and isolate the effective theory that describes the lightest fluctuations around this background. Motivated by the correspondence with the QHE [5], the gauge field we choose to turn on is a lift of the Yang monopole [6]. As we will see, this procedure leads to an hCS theory defined on fuzzy twistor space, coupled to fundamental matter localized on the twistor lines. In what follows, we will view the hCS theory as a decoupled subsector of an open string theory.

Consider hCS theory with gauge group $G = U(nN_c)$ where $n \equiv N+1$. We switch on a $U(1) \times SU(n)$ gauge field \mathcal{A}_Y which breaks the gauge group G to $U(N_c)$. \mathcal{A}_Y is obtained from the Yang monopole

$$\mathcal{A}_Y = \frac{1}{2}(dQ_a^\dagger Q_a - Q_a^\dagger dQ_a) \quad (16)$$

with Q_a defined in (9), via the replacement [5]

$$1 \rightarrow N\mathbf{1} \quad i \rightarrow -2i\mathbf{I}_1 \quad j \rightarrow -2i\mathbf{I}_2 \quad k \rightarrow -2i\mathbf{I}_3 \quad (17)$$

with $\mathbf{1}$ the $U(1)$ generator and \mathbf{I}_a the spin $N/2$ generators of $SU(2)$. The non-abelian part of \mathcal{A}_Y is trivial along the \mathbb{CP}^1 fibers, and maps via the twistor correspondence to a homogeneous $SU(n)$ instanton on S^4 with instanton number $k_{\text{inst}} = n(n^2 - 1)/6$. The abelian component of \mathcal{A}_Y is non-trivial along the \mathbb{CP}^1 fiber direction: each twistor line carries N units of $U(1)$ flux.

Let us consider the low energy fluctuations of the hCS theory around this background. To organize the low energy content, it is helpful to translate the bundle data into the language of D-branes. To avoid clutter, we take $N_c = 1$; the generalization to $N_c > 1$ is straightforward.

Twistor space contains \mathbb{CP}^p for $p \leq 3$ as a $2p$ -cycle. Brane bound states are therefore labeled by charge vectors $Q = (q_6, q_4, q_2, q_0)$ with q_p the Dp -brane wrapping number around the p -cycle. $U(n)$ hCS theory in the \mathcal{A}_Y background can be viewed as the world volume theory on a bound state of n space-filling $D6$ -branes with k $D2$ -branes wrapping the \mathbb{CP}^1 , with k given by eq. (13). In addition, the non-zero $U(1)$ flux through \mathbb{CP}^1 induces on each $D6$ an opposite $D4$ charge, while each $D2$ acquires an opposite $D0$ charge. The total brane configuration has a charge vector $Q = (n, -n, -k, k)$, and naturally splits up as an n -stack of $D6/D4$ -branes with charge vector $(n, -n, 0, 0)$ and a k -stack of $D2/D0$ -branes with charge vector $(0, 0, -k, k)$. The open string ground states thus organize into matrices of four different sizes

$$X_{k \times k}, B_{n \times k}, C_{k \times n}, Y_{n \times n} \quad (18)$$

X describes the collective motion of the $D2/D0$ -branes, and Y denotes the gauge and adjoint matter fields on the $D6/D4$ -branes. B and C are the open strings that live on the intersections between the two stacks of branes.

An important consequence of the extra $U(1)$ flux is that it lifts the degeneracy of the ADHM moduli space. Let V_6 and V_2 denote the bundles associated with the $D6/D4$ n -stack and $D2/D0$ k -stack. We can count the deformations of the bound state bundle via

$$\begin{aligned} \chi(V_6, V_2) &= \int \text{ch}(V_6^\vee) \text{ch}(V_2) \text{Td}(\mathbb{CP}^3) \\ &= 2kn - kn - kn = 0, \end{aligned}$$

where we used that $\text{Td}(\mathbb{CP}^3) = 1 + 2H + \dots$ with H the hyperplane class divisor, representing the \mathbb{CP}^2 4-cycle. The physical mechanism that lifts the moduli is that in terms of the $U(n)$ gauge theory on the $D6/D4$ -brane, the $U(1)$ flux represents a non-trivial vev for the adjoint scalars. This moves the $D6/D4$ -brane gauge theory slightly onto its Myers branch. The ADHM moduli still survive as the relevant low energy degrees of freedom, but they are no longer exact zero modes.

Twistor Matrix Model

Let us summarize our proposal. The effective theory of the light fluctuations of the $U(nN_c)$ hCS theory in the \mathcal{A}_Y background, we propose, takes the form of a $U(N_c)$ hCS theory on a fuzzy $\mathbb{CP}^{3|4}$ with a k dimensional Hilbert space $\mathcal{H}_{\mathbb{PT}}(N)$ with $n = N + 1$. In addition, the theory contains bifundamentals B and C connecting the k stack of $D2/D0$ -branes to the n stack of $D6/D4$ -branes. From the open string perspective, the fuzzification arises because the open string end points are charged under the background flux. The presence of the flux introduces an explicit choice of scale via $\text{Vol}(\mathbb{PT}) = \hbar^3 \dim \mathcal{H}_{\mathbb{PT}}$, and the open string spectrum is gapped, with excitations of energy $\hbar^{-1/2} \sim N^{1/2} \times \text{Vol}(\mathbb{PT})^{-1/6}$. The excitations decouple at large N , and in the low energy limit the modes

are forced to lie in the LLL. We will now construct the action for this low energy effective theory (see also [7]).

Given a commutative theory on a toric space, there exists a natural way to fuzzify the theory, while leaving the holomorphic geometry intact [3]. Here we apply this technology to the twistor gauge theory.

The hCS sector on fuzzy \mathbb{PT} captures the collective dynamics of the $D0$ -branes, described by the $k \times k$ matrix X in eq. (18). It consists of a $(0, 1)$ -form superfield $\mathcal{A} = \mathcal{A}_I(Z^\dagger, Z, \psi^\dagger, \psi) d\bar{Z}^I$, subject to the homogeneity constraint $[\mathfrak{D}, \mathcal{A}_I] = 0$, with \mathfrak{D} given in eq. (11).

Given an element $h(Z^\dagger, Z, \psi^\dagger, \psi)$ with coefficients in the Lie algebra $u(N_c)$, a gauge transformation on \mathcal{A}_I is given by $\mathcal{A} \rightarrow e^{-h} * \mathcal{A} * e^h + e^{-h} * \bar{\partial} e^h$. Here $\bar{\partial}$ is the Dolbeault operator on fuzzy $\mathbb{C}^{4|4}$, which acts by commutation with the holomorphic coordinates. The holomorphic Chern-Simons functional is $hCS(\mathcal{A}) = \text{Tr}_{u(N_c)} (\mathcal{A} \wedge * \bar{\partial} \mathcal{A} + \frac{2}{3} \mathcal{A} \wedge * \mathcal{A} \wedge * \mathcal{A})$. The action is then:

$$\mathcal{S}_{hCS} = \frac{1}{g_{\text{MM}}^2} \text{Tr}_{\mathcal{H}_{\mathbb{PT}}} (\Omega \wedge * hCS(\mathcal{A})) \quad (19)$$

where $\Omega = \frac{\varepsilon_{ijkl}}{4!} Z^I dZ^J dZ^K dZ^L \frac{\varepsilon^{ijkl}}{4!} \psi_i^\dagger \psi_j^\dagger \psi_k^\dagger \psi_l^\dagger$ and the trace over $\mathcal{H}_{\mathbb{PT}}$ is over all fuzzy points of the superspace consistent with the homogeneity constraint. The presence of the ψ^\dagger 's in the measure factor ensures the analogue of the relation $\int d^2\psi \bar{\psi} f(\psi) = \int d\psi f(\psi)$.

The B and C modes are (transposes of) linear maps $\mathcal{H}_{\mathbb{PT}} \rightarrow \mathcal{H}_{\mathbb{CP}^1}$. Geometrically, we can view these modes as localized on $D2/D6$ -brane intersections along the twistor lines. Just as for other intersecting brane configurations with localized modes, we treat them as bulk $(0, 1)$ forms with support along an appropriate subspace. The interaction terms between the gauge field \mathcal{A} and the B and C modes are found by expanding a parent $U(k+n)$ hCS theory around a breaking pattern to $U(k) \times U(n)$. The off-diagonal modes are the B, C fields, with action:

$$\mathcal{S}_{BC} = \text{Tr}_{\mathcal{H}_{\mathbb{CP}^1}} (B \cdot \bar{\partial}_{\mathbb{PT}} \cdot C) - \text{Tr}_{\mathcal{H}_{\mathbb{PT}}} (C \cdot \bar{\partial}_{\mathbb{CP}^1} \cdot B). \quad (20)$$

Here, $\bar{\partial}_{\mathbb{PT}} = \Omega \wedge * (\bar{\partial}_{\mathbb{PT}} + \mathcal{A})$ is the covariant derivative acting by matrix multiplication on $\mathcal{H}_{\mathbb{PT}}$: B and C are in the fundamental, rather than the adjoint. Similarly, $\bar{\partial}_{\mathbb{CP}^1}$ is a covariant derivative along the fuzzy \mathbb{CP}^1 fiber. (In the second term, some of the ‘‘bulk’’ form content of the B and C modes has been absorbed into $\bar{\partial}_{\mathbb{CP}^1}$.) The derivatives $\bar{\partial}_{\mathbb{PT}}$ and $\bar{\partial}_{\mathbb{CP}^1}$ correspond respectively to the vevs of the matrices $X_{k \times k}$ and $Y_{n \times n}$ in (18).

The parameters of the matrix model are the flux quanta N , and a continuous parameter g_{MM} as in eq. (19). Indeed, although the B-model is independent of Kähler data in the sense that we can always rescale Ω , once we introduce an explicit choice of flux and accompanying scale, this rescaling corresponds to adjusting a dilaton g_{MM} . The continuum limit is given by taking $N \rightarrow \infty$ with effective volume $\hbar^3 \dim \mathcal{H}_{\mathbb{PT}}$ held fixed. At large N , perturbation theory of the matrix model organizes according to the 't Hooft coupling $\lambda = g_{\text{MM}}^2 n N_c$.

Amplitudes and Continuum Limit

Natural observables of the twistor matrix model correspond to vevs involving localized versions of the B and C modes. The analogue of maximally helicity violating (MHV) amplitudes are obtained by focussing on the B, C correlators on a single twistor line \mathcal{H}_x . Define $B_x = B \cdot P_x$ and $C_x = P_x \cdot C$ with P_x the projection $\mathcal{H}_{\mathbb{P}^1} \rightarrow \mathcal{H}_x$. For normalized states $|z\rangle \in \mathcal{H}_x$, we can define the currents

$$J_x(z) = C_x |z\rangle \langle z| B_x \quad (21)$$

and write the analogue of MHV amplitudes as a matrix model vev:

$$A_{\text{MHV}}(z_i) = \int d^{4|8}x \left\langle \text{Tr}(J_x(z_1) \dots J_x(z_n)) \right\rangle_{\text{MM}} \quad (22)$$

where the trace is over \mathcal{H}_x and the color indices.

B_x and C_x are maps from $\mathcal{H}_{\mathbb{CP}^1}$ to \mathcal{H}_x and vice versa. So they act as $n \times n$ matrices, or equivalently, degree N homogeneous functions of the projective coordinates $u, v, u^\dagger, v^\dagger$ on a fuzzy $\mathbb{CP}^1 = \mathcal{H}_x$. The kinetic operator $\overline{D}_{\mathbb{P}^1}$ on the full twistor space, when sandwiched between B_x and C_x , thus naturally restricts to act only along the fiber. Inserting $B = B_x$ and $C = C_x$, the matrix model action reduces to a trace over the twistor line:

$$\mathcal{S}_{BC}(B_x, C_x) = \text{Tr}_{\mathbb{CP}^1}(B_x (\overline{\partial}_{\bar{z}} + \mathcal{A}_{\bar{z}}) C_x) \quad (23)$$

where now the Dolbault operator $\overline{\partial}_{\bar{z}}$ is defined by a commutator with the holomorphic coordinates along the fuzzy sphere. All objects in eq. (23) act as $n \times n$ matrices. At large N , this action tends to that of a commutative bc system with action $\int d^2z b_x \overline{\partial}_{\mathcal{A}}|_X c_x$ on a commutative twistor line X . For states $|z\rangle, |w\rangle \in \mathcal{H}_x$ associated with points z and w on \mathbb{CP}^1 , the large N vev of $\langle z|B_x C_x|w\rangle \sim \frac{1}{z-w}$. So doing the $n \times n$ matrix integral over B_x and C_x , while taking the large N limit, reproduces the correlators of the continuum bc system, which are known to generate the MHV amplitudes of $\mathcal{N} = 4$ SYM theory [8].

In the above argument, we assumed that the B_x, C_x matrix integral neatly separates from the modes at other locations along the S^4 . We will now argue that this approximation is justified at large N . The number of twistor lines $\mathcal{H}_{x_1}, \dots, \mathcal{H}_{x_d}$ necessary to span the bosonic part of $\mathcal{H}_{\mathbb{P}^1}$ is $(N+1)(N+2)/2$. Perform a decomposition $B = \sum_i B_i \cdot P_i$ and $C = \sum_j P_j \cdot C_j$, where P_i is the projection operator on the twistor line \mathcal{H}_{x_i} . For sufficiently well separated x_i and x_j , the action $\mathcal{S}_{BC}(B_i, C_j)$ is highly suppressed at large N . The intuitive reason for this localization is that the B and C modes are confined to small Landau orbits of angular size $1/\sqrt{N}$ (see eq. (15)), due to the presence of the large background flux. Note that the kinetic operator $\overline{D}_{\mathbb{P}^1}$ acts in the fundamental representation along the S^4 , so all B and C modes are sensitive to the flux. So we see that for large N , the

B, C matrix model essentially factorizes into a collection of decoupled BC systems $\mathcal{S}_{BC}(B_i, C_i)$.

These BC systems are still all coupled to the bulk field \mathcal{A} , which propagates between the MHV vertices. In the large N limit this therefore reproduces the main features of the CSW relations [9], provided we identify:

$$\lambda = g_{\text{MM}}^2 n N_c = g_{\text{YM}}^2 N_c. \quad (24)$$

In this way, the matrix model naturally makes contact with the twistor gauge theory proposed in [11], based on the MHV generating function.

Conclusion

Fuzzy twistor space provides a novel covariant regulator for gauge theory. Bulk space-time physics is dual to a large N matrix model obtained as a limit of holomorphic Chern-Simons theory in the presence of a Yang monopole. The background flux introduces an explicit length scale which demarcates the breakdown of UV locality. Though the twistor matrix model shares many ingredients familiar from the twistor string theory of [2], we expect the coupling to a closed string sector to evade the pathologies of conformal gravity [13]. Rather, it is natural to conjecture that the UV cutoff should be identified with the Planck scale [12].

Acknowledgements: We thank N. Arkani-Hamed, R. Boels, J. Broedel, S. Caron-Huot, D. Gross, T. Hartman, J. Maldacena, E. Silverstein, D. Skinner, L. Susskind, E. Verlinde, E. Witten and M. Yamazaki for helpful discussions. The work of JJH is supported by NSF grant PHY-0969448. The work of HV is supported by NSF grant PHY-0756966.

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